Metric Unification of Gravitation and Electromagnetism Solves the Cosmological Constant Problem

Murat Özer
Department of Physics, College of Science, King Saud University,
P. O. Box 2455, Riyadh 11451, Saudi Arabia

February 7, 2008

Abstract

We first review the cosmological constant problem, and then mention a conjecture of Feynman according to which the general relativistic theory of gravity should be reformulated in such a way that energy does not couple to gravity. We point out that our recent unification of gravitation and electromagnetism through a symmetric metric tensor has the property that free electromagnetic energy and the vacuum energy do not contribute explicitly to the curvature of spacetime just like the free gravitational energy. Therefore in this formulation of general relativity, the vacuum energy density has its very large value today as in the early universe, while the cosmological constant does not exist at all.

The mysterious cosmological constant[1] has been with us for more than eighty years ever since Einstein introduced it in 1917 [5] to obtain a static universe by modifying his field equations to

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \lambda_b g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1}$$

where λ_b is the (bare) cosmological constant, $R_{\mu\nu}$ is the Ricci tensor, $R = R^{\mu}_{\mu}$ is the curvature scalar, G is Newton's gravitational constant and c is the speed of light. Here $T_{\mu\nu}$ is the energy-momentum tensor which represents the energy content of space to which all forms of energy such as matter energy, radiation energy, electromagnetic energy, thermal energy, etc. contribute. Gravitational energy, however, is not included in $T_{\mu\nu}$ because it is already included (implicitly) on the left of

[1] See references [1] for a nontechnical and [2, 3, 4] for technical reviews.

eq.(1). For a homogeneous and isotropic universe $T_{\mu\nu}$ necessarily takes the form of the energy-momentum tensor of a perfect fluid

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) U_\mu U_\nu + \frac{p}{c^2} g_{\mu\nu}, \tag{2}$$

where ρ, p, and U_μ are respectively the energy density, pressure and four-velocity of the fluid. When the field equations (1) are applied to the Robertson-Walker metrics

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{3}$$

the Friedmann equation ensues:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{c^4}\rho + \frac{\lambda_b c^2}{3} - \frac{kc^2}{a^2}, \tag{4}$$

where $a(t)$ is the scale factor of the universe and $k = -1, 0, +1$ for a universe that is respectively spatially open, flat, and closed.

Be it defined as 'empty space' or 'the state of lowest energy' of a theory of particles, the vacuum has a lot of energy associated with it. The energy of the vacuum per unit volume, the vacuum energy density, has numerous sources. Virtual fluctuations of each quantum field corresponding to a particle and the potential energy of each field contribute to it. Stipulating that the vacuum be Lorentz invariant entails the energy-momentum tensor for it to be

$$T^{vac}_{\mu\nu} = -\rho_{vac} g_{\mu\nu}, \tag{5}$$

because $g_{\mu\nu}$ is the only 4×4 tensor that is invariant under Lorentz transformations. Comparing eq.(5) with eq.(2) reveals that the vacuum has the equation of state $p_{vac}/c^2 = -\rho_{vac}$ [6]. According to our current understanding, the value of the density of energy that resides in the vacuum has no relevance in nongravitational physics both at the classical and quantum levels. However, being a form of energy density, ρ_{vac} takes its place in the field equations (1), thus modifying $T_{\mu\nu}$ to

$$T_{\mu\nu} = T^M_{\mu\nu} + T^{vac}_{\mu\nu}, \tag{6}$$

where now $T^M_{\mu\nu}$ is the total energy-momentum tensor of the space other than that of the vacuum. With $T^{vac}_{\mu\nu}$ included, the Friedmann equation (4) changes to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{c^4}\rho + \frac{8\pi G}{c^4}\rho_{vac} + \frac{\lambda_b c^2}{3} - \frac{kc^2}{a^2}, \tag{7}$$

from which the 'effective cosmological constant' is defined to be

$$\begin{aligned} \lambda^{eff} &= \lambda_b + \lambda_{vac} \\ &= \lambda_b + \frac{8\pi G}{c^4}\rho_{vac} \\ &= \frac{8\pi G}{c^4}\left(\frac{c^4 \lambda_b}{8\pi G} + \rho_{vac}\right) \\ &= \frac{8\pi G}{c^4}\rho^{eff}_{vac}. \end{aligned} \tag{8}$$

This means that even if the bare cosmological constant λ_b is zero, the effective cosmological constant is not. Anything that contributes to the energy density of the vacuum is tantamount to a cosmological constant. The present value of ρ_{vac}^{eff} can be estimated from astronomical observations. The Hubble constant at the present epoch is $H_0 = (\dot{a}/a)_0 = 50 - 80\ kms^{-1}Mpc^{-1}$. The critical energy density of the universe, ρ_c, in the absence ov ρ_{vac}^{eff} is defined by

$$\rho_c = \frac{3H_0^2c^2}{8\pi G} \tag{9}$$

and has the value $4.7\times10^{-27}kgm^{-3}\,c^2 = 2\times10^{-47}GeV^4$ for $H_0 = 50kms^{-1}Mpc^{-1}$. In terms of the present values of the density parameters $\Omega_0 = \rho_0/\rho_c$ and $\Omega_{vac}^{eff} = \rho_{vac}^{eff}/\rho_c$ eq.(7) can be cast into

$$1 = \left(\Omega_0 + \Omega_{vac}^{eff} - \frac{kc^2}{a_0^2H_0^2}\right). \tag{10}$$

Since no effects of the spatial curvature are seen, the curvature term in eq.(10) can be safely neglected. Observations give us that $\Omega_0 = 0.1 - 0.4$, with dark matter included, from which $\Omega_{vac}^{eff} \approx 0.6 - 0.9$ follows from eq.(10). Hence, we conclude that today

$$\begin{aligned} \rho_{vac}^{eff} &< \rho_c \approx 10^{-47}\,GeV^4 \\ \lambda^{eff} &< 10^{-52}\,m^{-2}. \end{aligned} \tag{11}$$

As well known, this is in great disagreement with the values predicted by gauge field theories, of which the best example and the experimentally well established one is the electroweak theory [7, 8, 9]. In this theory, as a result of spontaneous symmetry breaking with a Higgs doublet ϕ, the minimum of the potential of ϕ contributes to the vacuum energy density and hence to the cosmological constant by [10, 11]

$$\begin{aligned} \mid \rho_{vac} \mid &= \mid V_{min} \mid = \mid -\frac{1}{8}M_H^2v^2 \mid \approx 2\times10^8\,GeV^4 \\ \mid \lambda_{vac} \mid &= \frac{8\pi G}{c^4} \mid \rho_{vac} \mid \approx 10^3\,m^{-2}, \end{aligned} \tag{12}$$

where it is assumed that the potential vanishes at $\phi = 0$, and a Higgs particle of mass $M_H \approx 150\,GeV$ together with $v = 246\,GeV$ have been used. These are larger than the present bounds on ρ_{vac}^{eff} and λ^{eff} by a factor of 10^{55}. The two terms in $\rho_{vac}^{eff} = c^4\lambda_b/8\pi G + \rho_{vac}$ must cancel to at least 55 decimal places so as to reduce ρ_{vac}^{eff} and hence λ^{eff} to their small values today. So, in the context of Einstein's general relativity theory, the cosmological constant problem is to understand through what natural mechanism the vacuum energy density ρ_{vac} got reduced to its small value today. It is not why it was always small. Many different approaches to the problem, none being entirely satisfactory, have been tried [2]. The idea which triggered the

solution that we shall present here belongs to Feynman. In an interview on Superstrings, while talking about gravity he said [12]: 'In the quantum field theories, there is an energy associated with what we call the vacuum in which everything has settled down to the lowest energy; that energy is not zero-according to the theory. Now gravity is supposed to interact with every form of energy and should interact then with this vacuum energy. And therefore, so to speak, a vacuum would have a weight-an equivalent mass energy-and would produce a gravitational field. Well, it doesn't! The gravitational field produced by the energy in the electromagnetic field in a vacuum-where there's no light, just quiet, nothing-should be enermous, so enermous, it would be obvious. The fact is, it's zero! Or so small that it's completely in disagreement with what we'd expect from the field theory. This problem is sometimes called the cosmological constant problem. It suggests that we're missing something in our formulation of the theory of gravity. It's even possible that the cause of the trouble-the infinities-arises from the gravity interacting with its own energy in a vacuum. And we started off wrong because we already know there's something wrong with the idea that gravity should interact with the energy of a vacuum. So I think the first thing we should understand is how to formulate gravity so that it doesn't interact with the energy in a vacuum. Or maybe we need to formulate the field theories so there isn't any energy in a vacuum in the first place.'

The purpose of this letter is to point out that (i) such a formulation of gravity in which the free electromagnetic and vacuum energy does not disturb the emptiness of space, just like the free gravitational energy, in the sense that their effects are already implicitly included on the left side of the field equations has recently been formulated by us [13, 14] and (ii) there does not exist a cosmological constant problem in this formulation. This new formulation, however, is not only a formulation of gravity but also a unified description of gravity and electromagnetism through a symmetric metric tensor. The impossibility of describing the motion of charged particles with different charge-to-mass ratios in an electromagnetic field by a single geometry leads us to consider classes of geometries corresponding to different charge-to-mass ratios. This way the electromagnetic force on a given charged test particle can be geometrized [13]. Consider an object with a distribution of charged matter with total mass M_o and charge Q_o. Let there be another distribution of matter M and charge Q external to the object. Let also a test particle of mass m and charge q be moving in the charge distribution external to the object. The modified field equations are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T^{M}_{\mu\nu} + \frac{k_e}{c^4}\frac{q}{m}T^{CC}_{\mu\nu}, \tag{13}$$

as opposed to the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\left[T^{M}_{\mu\nu} + T^{EM}_{\mu\nu}(Q) + T^{EM}_{\mu\nu}(Q_o) + T^{vac}_{\mu\nu}\right] \tag{14}$$

where k_e is the (Coulomb) electric constant, $T^{M}_{\mu\nu}$ is the matter energy-momentum tensor of the distribution outside the object. $T^{EM}_{\mu\nu}(Q_o)$ and $T^{EM}_{\mu\nu}(Q)$ are the energy-

momentum tensors due to the electromagnetic fields of the object and the charge distribution outside it, respectively. $T_{\mu\nu}^{CC}$ is the so called charged-current tensor, given by

$$T_{\mu\nu}^{CC} = \frac{1}{3} v_\alpha \mathcal{J}^\alpha \left(\frac{1}{c^2} U_\mu U_\nu + g_{\mu\nu} \right), \tag{15}$$

where $v^\alpha = (\gamma_v c, \gamma_v \vec{v})$ is the four velocity of the test particle, $\mathcal{J}^\alpha = (c\rho_Q, \vec{J} + \vec{J}_D)$ with ρ_Q, $\vec{J}$, and $\vec{J}_D$ being respectively the charge density, current density, and the displacement current density of the charge distribution outside the object, $U^\mu = (\gamma_u c, \gamma_u \vec{u})$ is the four-velocity of the charge distribution, and $\gamma_{v(u)} = (1 - v^2(u^2)/c^2)$. $T_{\mu\nu}^{CC}$ is not an energy momentum tensor. The right-hand side of eq.(13) does not contain the energy- momentum tensor of the electromagnetic field due to the charge distribution of the object. For example, the field equations describing a spherical distribution of mass M and charge Q located at $r = 0$ are

$$R_{\mu\nu} = 0 \tag{16}$$

and has the solution

$$ds^2 = -\left(1 - 2\frac{GM}{c^2 r} + 2\frac{q}{m}\frac{k_e Q}{c^2 r}\right) c^2 dt^2 + \left(1 - 2\frac{GM}{c^2 r} + 2\frac{q}{m}\frac{k_e Q}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2. \tag{17}$$

Whereas in Einstein's general relativity, one has

$$R_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}^{EM} \tag{18}$$

instead of eq.(15), where $T_{\mu\nu}^{EM}$ is the energy-momentum tensor of the electromagnetic field of the charged sphere. Einstein's general relativity is a theory of gravitation. Our general relativity is a theory of gravitation and electromagnetism. As well known, the solution of eq.(17) is the Reissner-Nordstrøm solution

$$ds^2 = -\left(1 - 2\frac{GM}{c^2 r} + \frac{G k_e Q^2}{c^4 r^2}\right) c^2 dt^2 + \left(1 - 2\frac{GM}{c^2 r} + \frac{G k_e Q^2}{c^4 r^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2, \tag{19}$$

To sum up, this new formulation of general relativity describes gravitation and electromagnetism as an effect of the curvature of spacetime produced by matter energy and electric charge. We have suggested a very simple deflection of electrons by a spherical charge distribution experiment in ref.[14] to distinguish between the two theories.

Having presented the salient features of our formulation, we can now present our solution to the cosmological constant problem: The vacuum energy today is as large as it can be. It has the same very large energy density today as it had in

the early universe. The contribution of the vacuum to the cosmological constant is simply zero. This is because no form of energy-momentum tensor except for that of the matter is allowed on the right side of the modified field equations (13). The effect on the curvature of spacetime of any form of free energy like gravitational, electromagnetic, and vacuum is already included (implicitly) on the left of eq.(13). In Einstein's general relativity the contribution of ρ_{vac} to λ^{eff} is $\lambda_{vac} = (8\pi G/c^4)\rho_{vac}$, whereas in our formulation $\lambda_{vac} = 0 \times \rho_{vac} = 0$. As for the bare cosmological constant λ_b in eq.(1), one may mathematically include it in our eq.(13). But it cannot have the physical meaning of a some sort of bare contribution to the effective vacuum energy density. If, somehow, there is an unknown bare contribution ρ_b to the (effective) vacuum energy density, then the corresponding bare cosmological constant is $\lambda_b = 0 \times \rho_b = 0$ in our formulation.

In conclusion, if a metric unification of gravitation and electromagnetism is realized in nature, then gravitation is no different from other interactions so far as the effects of the vacuum energy are concerned. The vacuum energy density and the cosmological constant are not related; the former has a very large value while the latter does not exist at all. Immediate performance of the experiment of the deflection of electrons by a positive spherical charge distribution [14] cannot be overemphasized.

References

[1] L. Abbott, Sci. Am. May 1988(1988)82.

[2] S. Weinberg, Rev. Mod. Phys. 61(1989)1.

[3] S. M. Carroll, W. H. Press, E. L. Turner, Annu. Rev. Astron. Astrophys. (1992)499.

[4] V. Sahni and A. Starobinsky, astro-ph/9904398.

[5] A. Einstein, Sitz. Preuss. Akad. Wiss. 142(1917).

[6] Ya. B. Zel'dovich, JETP Lett. 6(1967)316; Sov. Phys. Uspekhi 11(1968)381.

[7] S. L. Glashow, Nucl. Phys. 22(1961)579.

[8] S. Weinberg, Phys. Rev. Lett. 19(1967)1264.

[9] A. Salam, in Proceedings of the VIII Nobel Symposium, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p.367.

[10] J. Dreitlein, Phys. Rev. Lett. 33(1974)1243.

[11] A. D. Linde, JETP Lett. 19(1974)183.

[12] P. C. W. Davies and J. Brown (ed.), Superstrings, A Theory of Everything, (1988) Cambridge University Press, p.201.

[13] M. Özer, 'On the Equivalence Principle and a Unified Description of Gravitation and Electromagnetism', gr-qc/9910062.

[14] M. Özer, 'Proposed Experiments to Test the Unified Description of Gravitation and Electromagnetism Through a Symmetric Metric', gr-qc/9910095.